Series - Electronic and optical semiconductor devices

Series 9 - p-n junction out of equilibrium

Exercise

I - Elementary picture of rectification by a p-n junction

1. V_{bi} is the total electrostatic potential drop across the junction when $V = 0$. It results immediately that $\boxed{\Delta V = V_{bi} - V}$

2. Considering Series 8 where we established the expression of the width of the depleted region for an unbiased junction, we deduce that $W(V)$ will express as:

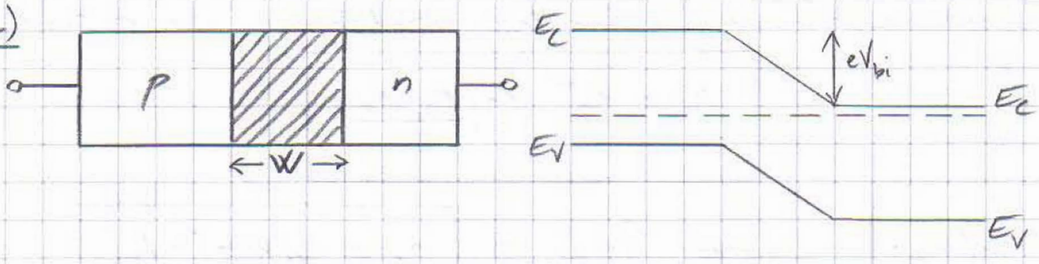
$$\boxed{W(V) = \left[\frac{2\varepsilon_r \varepsilon_0}{e}\left(\frac{N_A+N_D}{N_A N_D}\right)(V_{bi}-V)\right]^{1/2}}$$

This expression can be rewritten as a function of $W(0)$ which leads to:

$$W(V) = \underbrace{\left[\frac{2\varepsilon_r \varepsilon_0}{e}\left(\frac{N_A+N_D}{N_A N_D}\right)V_{bi}\right]^{1/2}}_{W(0)} \times \left[1 - \frac{V}{V_{bi}}\right]^{1/2}$$

i.e. $$\boxed{W(V) = W(0)\left[1 - \frac{V}{V_{bi}}\right]^{1/2}}$$

3- a)



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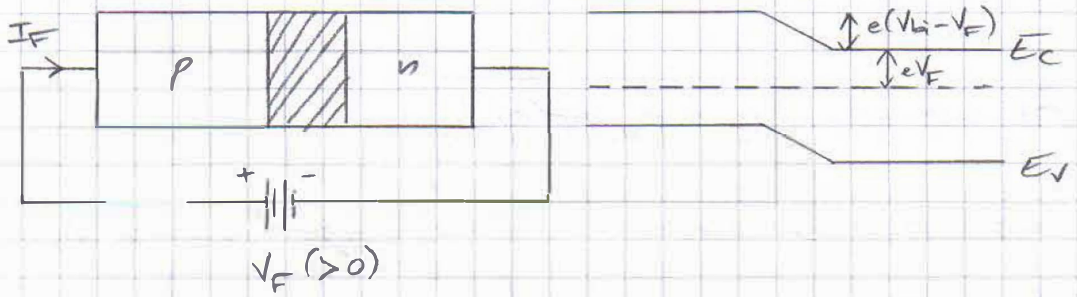

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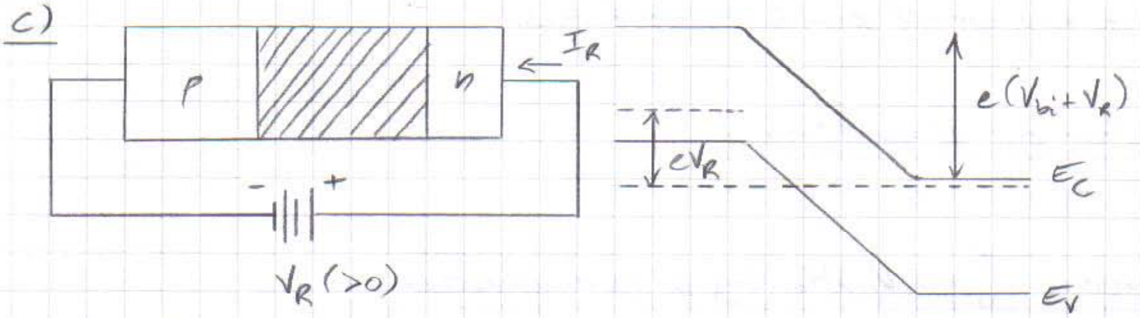

II - Ideal current-voltage characteristics of a p-n junction

4- Under forward bias, the applied voltage reduces the electrostatic potential drop across the junction. As a result, the drift current will decrease compared with the diffusion current i.e., the diffusion of holes from the p-side to the n-side will increase and vice-versa for the diffusion of electrons. We will thus have injection of minority carriers i.e. electrons are injected on the p-side and holes are injected on the n-side. Under reverse bias, the applied voltage increases the electrostatic potential drop across the junction which induces a significant decrease of diffusion currents therefore leading to a small reverse current.

5- When studying the p-n junction at equilibrium, it was shown that $V_{bi} = \frac{k_B T}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right)$. In addition, one of the assumptions considered to derive the ideal current-voltage characteristics is that outside the depletion region the semiconductor is assumed to be neutral which means we can write $N_A = p_{p_0}$ and $N_D = n_{n_0}$. Furthermore, the mass action law established on the p-side will write $p_{p_0} n_{p_0} = n_i^2$ so that $\boxed{V_{bi} = \frac{k_B T}{e} \ln\left(\frac{p_{p_0} n_{n_0}}{p_{p_0} n_{p_0}}\right)}$

i.e. $\boxed{V_{bi} = \frac{k_B T}{e} \ln\left(\frac{n_{n_0}}{n_{p_0}}\right)}$ which leads to $\boxed{n_{n_0} = n_{p_0} \exp\left(\frac{eV_{bi}}{k_B T}\right)}$

Similarly for holes we get $\boxed{p_{p_0} = p_{n_0} \exp\left(\frac{eV_{bi}}{k_B T}\right)}$.

6- Under forward bias ($V>0$), the electrostatic potential drop becomes $V_{bi} - V_F$ ($V_F > 0$) whereas under reverse bias the electrostatic potential drop increases and becomes $V_{bi} + V_R$ (a negative voltage $-V_R$ is applied to the p-n junction). As a result from the previous question, we can write for electrons that

$$\boxed{n_n = n_p \exp\left(\frac{e(V_{bi}-V)}{k_B T}\right)}$$

7- In the low injection condition, the injected minority carrier densities are small compared with the majority carrier densities so that $n_n \approx n_{n_0}$. As a result the electron density at the boundary of the depletion region on the p-side $(x=-x_p)$ will write:

$$n_{n_0} = n_p \exp\left(\frac{e(V_{bi}-V)}{k_B T}\right) \quad \text{i.e.} \quad n_p = n_{n_0} \exp\left(\frac{eV}{k_B T}\right) \exp\left(-\frac{eV_{bi}}{k_B T}\right)$$

and as $n_{n_0} = n_{p_0} \exp\left(\frac{eV_{bi}}{k_B T}\right)$ we obtain $\boxed{n_p = n_{p_0} \exp\left(\frac{eV}{k_B T}\right)}$

and $n_p - n_{p_0} = n_{p_0}\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$

Similarly, for holes we get $\boxed{p_n = p_{n_0} \exp\left(\frac{eV}{k_B T}\right)}$

and $p_n - p_{n_0} = p_{n_0}\left[\exp\left(\frac{eV}{k_B T}\right) - 1\right]$

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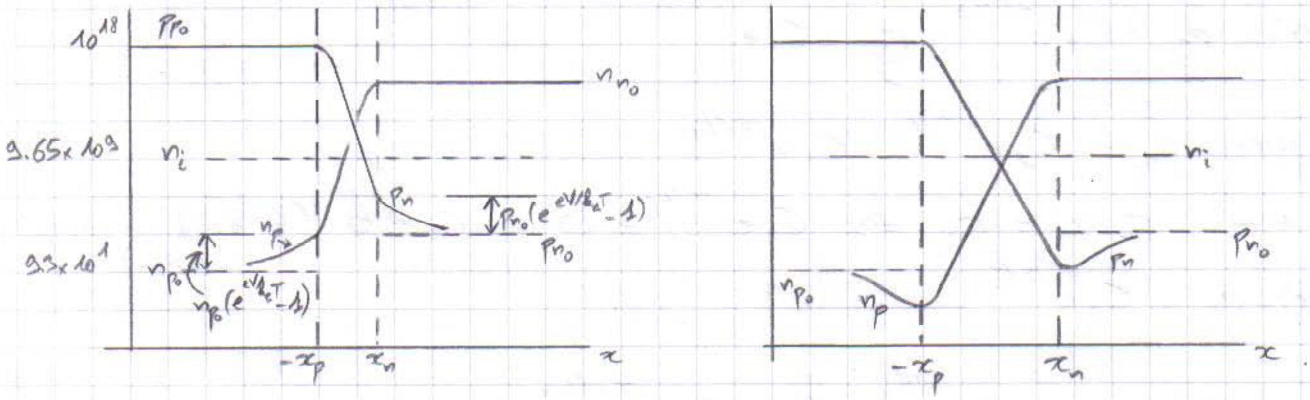

a- Forward bias

b. Reverse bias

The minority carrier densities at the boundaries of the depletion region (at $x=-x_p$ and x_n) are much larger than their equilibrium values under forward bias whereas they are lower than their equilibrium values under reverse bias.

9- $L_p = \sqrt{D_p \tau_p}$ corresponds to the hole diffusion length (minority carriers) on the n-side. The boundary conditions on the n-side are such that

$p_n(x=x_n) = p_{n_0} \exp\left(\frac{eV}{k_B T}\right)$ and $p_n(x=+\infty) = p_{n_0}$.

The solution to the steady-state continuity equation can be obtained by applying the method of the variation of $\left\{\begin{matrix}\text{constants}\\ \text{parameters}\end{matrix}\right.$ to a second order differential equation with a constant second member. This is done first by solving

$p_n'' - \frac{p_n}{L_p^2} = 0$ of characteristic equation $r^2 - \frac{1}{L_p^2} = 0$ i.e., $\left(r - \frac{1}{L_p}\right)\left(r + \frac{1}{L_p}\right) = 0$

We thus have $p_n(x) = \lambda(x) e^{-x/L_p} + \mu(x) e^{x/L_p}$

and $p_n'(x) = \lambda' e^{-x/L_p} - \frac{\lambda}{L_p} e^{-x/L_p} + \mu' e^{x/L_p} + \frac{\mu}{L_p} e^{x/L_p}$

where we set $\lambda' e^{-x/L_p} + \mu' e^{x/L_p} = 0$ (1)

$p_n''(x) = -\frac{\lambda'}{L_p} e^{-x/L_p} + \frac{\lambda}{L_p^2} e^{-x/L_p} + \frac{\mu'}{L_p} e^{x/L_p} + \frac{\mu}{L_p^2} e^{x/L_p}$

so that $-\frac{\lambda'}{L_p} e^{-x/L_p} + \cancel{\frac{\lambda}{L_p^2} e^{-x/L_p}} + \frac{\mu'}{L_p} e^{x/L_p} + \cancel{\frac{\mu}{L_p^2} e^{x/L_p}} - \cancel{\frac{\lambda}{L_p^2} e^{-x/L_p}} - \cancel{\frac{\mu}{L_p^2} e^{x/L_p}} = -\frac{p_{n_0}}{L_p^2}$

i.e. $-\lambda' e^{-x/L_p} + \mu' e^{x/L_p} = -\frac{p_{n_0}}{L_p}$ (2)

(1)+(2) $\Rightarrow 2\mu' e^{x/L_p} = -\frac{p_{n_0}}{L_p} \Rightarrow \mu' = -\frac{p_{n_0}}{2L_p} e^{-x/L_p}$ i.e. $\mu(x) = \frac{p_{n_0}}{2} e^{-x/L_p} + C_1$

From (1), $\lambda' = -\mu' e^{2x/L_p} \Rightarrow \lambda' = \frac{p_{n_0}}{2L_p} e^{x/L_p}$ i.e. $\lambda(x) = \frac{p_{n_0}}{2} e^{x/L_p} + C_2$

Then $p_n(x) = \frac{p_{n_0}}{2} + C_2 e^{-x/L_p} + \frac{p_{n_0}}{2} + C_1 e^{x/L_p}$

So that $p_n(x) - p_{n_0} = C_2 e^{-x/L_p} + C_1 e^{x/L_p}$

At $x = +\infty$, $p_n(x) = p_{n_0} \Rightarrow C_1 e^{+\infty/L_p} = 0 \Rightarrow C_1 = 0$ (to avoid an unphysical solution)

$\Rightarrow p_n(x) - p_{n_0} = C_2 e^{-x/L_p}$

At $x = x_n$, $p_n - p_{n_0} = p_{n_0}(e^{eV/k_BT} - 1) = C_2 e^{-x_n/L_p}$

$\Rightarrow C_2 = p_{n_0}(e^{eV/k_BT} - 1) e^{x_n/L_p}$

so that $\boxed{p_n(x) - p_{n_0} = p_{n_0}(e^{eV/k_BT} - 1) e^{-(x-x_n)/L_p}}$

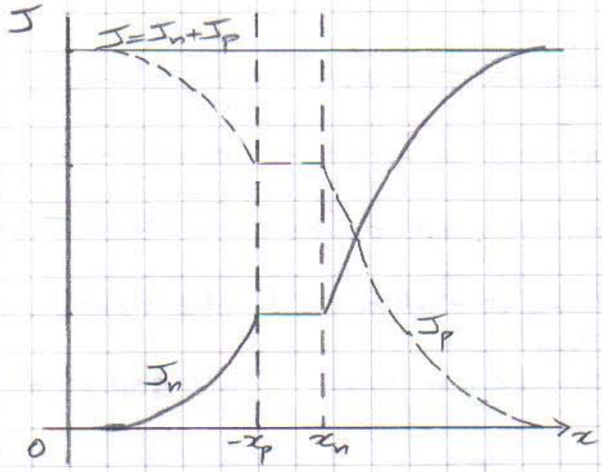
10- Outside the depletion region, the electric field is zero so that $J_p(x_n) = -eD_p \left.\frac{dp_n}{dx}\right|_{x=x_n}$

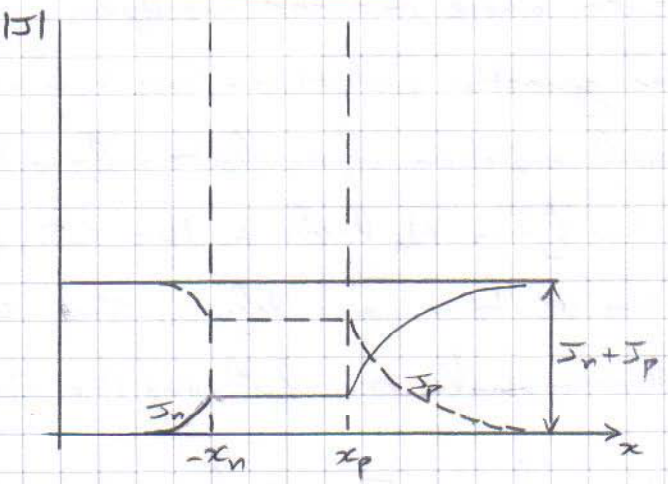
i.e. $\boxed{J_p = \frac{eD_p p_{n_0}}{L_p}(e^{eV/k_BT} - 1)}$

11- For electrons, we can establish the following equations which are valid in the neutral p-type region:

$$\boxed{\begin{aligned} n_p(x) - n_{p_0} &= n_{p_0}\left(e^{eV/k_BT} - 1\right) e^{(x+x_p)/L_p} \\ J_n &= \frac{eD_n n_{p_0}}{L_n}\left(e^{eV/k_BT} - 1\right) \end{aligned}}$$

12- a- Forward bias　　　　b- Reverse bias



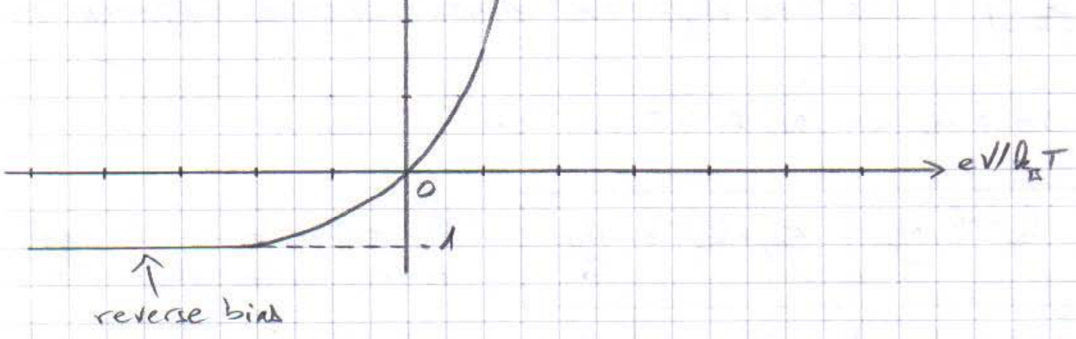


13- $J = J_p(x_n) + J_n(-x_p)$ (here we use the fact that the current associated with electrons and holes is constant throughout the depletion region) so that:

$$\boxed{J = \frac{e D_p p_{n_0}}{L_p} + \frac{e D_n n_{p_0}}{L_n}\left(e^{eV/k_BT} - 1\right)} \quad \text{and} \quad \boxed{J_s = \frac{e D_p p_{n_0}}{L_p} + \frac{e D_n n_{p_0}}{L_n}}$$

represents the saturation current density i.e., the current flowing through the junction under reverse bias. The main aspect when treating the ideal p-n junction is that its characteristics are governed by the diffusion of minority carriers.

14-



15- $J_s = \frac{e D_p p_{n_0}}{L_p} + \frac{e D_n n_{p_0}}{L_n} = e n_i^2\left[\frac{1}{N_D}\left(\frac{D_p}{\tau_p}\right)^{1/2} + \frac{1}{N_A}\left(\frac{D_n}{\tau_n}\right)^{1/2}\right] \approx 8.85\times10^{-12}\,A.cm^{-2}$

and $I_s = J_s \times A = 1.72\times10^{-15}\,A$

Additional remarks

* $V \ll V_{bi}$ (which is readily seen from question 2). To be more quantitative, I drastically increases once eV exceeds the thermal energy $k_B T$ (cf. questions 13 and 14). Usually we consider a device operating at room temperature i.e. a situation where impurities are fully ionized.

* The generation current can have two main origins. The first one is the thermal excitation of an electron from the valence band to the conduction band (or from N_A levels to the CB, from midgap defects to the CB or from the VB to midgap defects). The second one is linked with the creation of an electron-hole pair following the absorption of an above band gap photon.

The notion of quasi-Fermi levels and the mass action law

The electron density per unit volume n is given by the overlap between the Fermi-Dirac distribution and the density of states per unit volume in the conduction band (CB) i.e.

$$n = \int_{E_c}^{+\infty} \rho_c(E) f(E)\, dE \quad \text{with } \rho_c(E) = \frac{V}{2\pi^2}\left(\frac{2m_c^*}{\hbar^2}\right)^{3/2}(E-E_c)^{1/2} \text{ (3D case, parabolic approximation)}$$

$$f(E) = [1 + \exp(E-E_F)/k_B T]^{-1}$$

Case 1: unoccupied Fermi level

The Fermi level is sufficiently deep into the band gap vs. $k_B T$ to fulfill the condition $(E-E_F) \gg k_B T \Rightarrow$ Fermi-Dirac distribution $\simeq$ Boltzmann distribution (classical regime) so that $n = \int_{E_c}^{+\infty} \rho_c(E) \exp[-(E-E_F)/k_B T]\, dE = N_c \exp[-(E_c - E_F)/k_B T]$

and $N_c = \frac{1}{4}\left(\frac{2 m_c^* k_B T}{\pi \hbar^2}\right)^{3/2}$ is the effective density of states (effective DOS)

For GaAs, $m_c^* = 0.067\, m_0 \Rightarrow N_c \approx 4.3 \times 10^{17}\, \text{cm}^{-3}$ at 300 K

$$\Rightarrow \boxed{E_F = E_c - k_B T \ln \frac{N_c}{n} = E_V + k_B T \ln \frac{N_V}{p}}$$ (the same treatment applies to the case of holes)

As seen in an earlier series, the Fermi level goes into the bands i.e., the semiconductor is said to be degenerate, once the electron or hole density is superior to the effective DOS.

Mass action law

When electrons and holes are at thermal equilibrium they share the same Fermi level (chemical potential) so that $np = N_C N_V \exp(-E_g/k_B T)$ and for an undoped semiconductor $n = p = n_i = (N_C N_V)^{1/2} \exp(-E_g/2k_B T)$.

For a doped semiconductor (we consider a donor density N_D), assuming that at the temperature of interest donor atoms are fully ionized and if $N_D \gg n_i$, we can show that free carriers in the CB have all been thermally excited from donor levels so that $n = N_D$ and $\boxed{E_F = E_C - k_B T \ln \frac{N_C}{N_D}}$

Similarly for acceptors we get $p = N_A$ and $\boxed{E_F = E_V + k_B T \ln \frac{N_V}{N_A}}$

Case 2: Occupied Fermi level (degenerate case)

In that case, we perform the approximation: $f(E) = 1$ if $E < E_F$

$f(E) = 0$ if $E > E_F$

$$\Rightarrow n = \int_{E_C}^{E_F} \rho_C(E)\,dE = \frac{1}{3\pi^2}\left(\frac{2m_C^*}{\hbar^2}\right)^{3/2}(E_F - E_C)^{3/2}$$ (simplified approach vs. Sommerfeld expansion!)

↳ metallic behavior, E_F is independent of the temperature and it is occupied even at 0 K.

Quasi-Fermi levels in a system out of equilibrium

① External generation process of electrons and holes $\Rightarrow$ generation rate G_n and G_p (in $cm^{-3}s^{-1}$).

② Recombination process with time constants τ_n and τ_p leading to steady-state electron and hole populations given by the equality between generation and recombination processes i.e. $G_n = \frac{n}{\tau_n}$ and $G_p = \frac{p}{\tau_p}$

The hypothesis made when establishing the previous equations is that the generated carriers (either by photon absorption, electrical injection...) are present in densities which exceed by far thermal densities i.e $n \geq n_0$ and p_0 (where n_0 and p_0 where denoted n and p in the first part of this section).

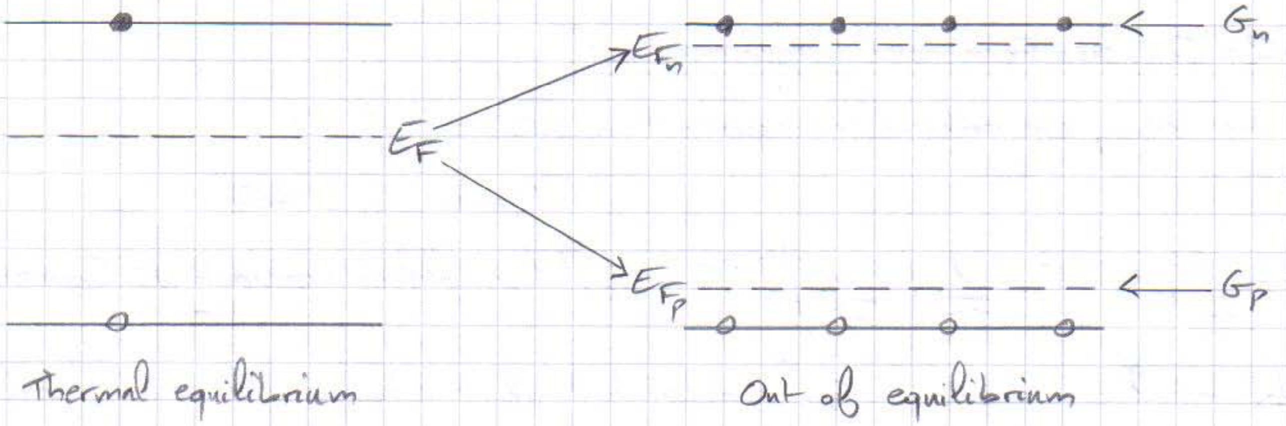
The concept introduced by William B. Shockley (Nobel Prize in Physics in 1956 for the joint discovery with John Bardeen and Walter H. Brattain of the transistor effect) consists in considering that those free carrier populations can still be described by quasi-Fermi levels.

If the system is non degenerate, we get

$$\boxed{\begin{aligned} E_{F_n} &= E_C - k_B T \ln\left(\frac{N_C}{G_n \tau_n}\right) \\ E_{F_p} &= E_V + k_B T \ln\left(\frac{N_V}{G_p \tau_p}\right) \end{aligned}}$$

If the system is degenerate, we get

$$\boxed{\begin{aligned} E_{F_n} &= E_C + \frac{\hbar^2}{2m_C^*}(3\pi^2 n)^{2/3} \\ E_{F_p} &= E_V - \frac{\hbar^2}{2m_V^*}(3\pi^2 p)^{2/3} \end{aligned}}$$

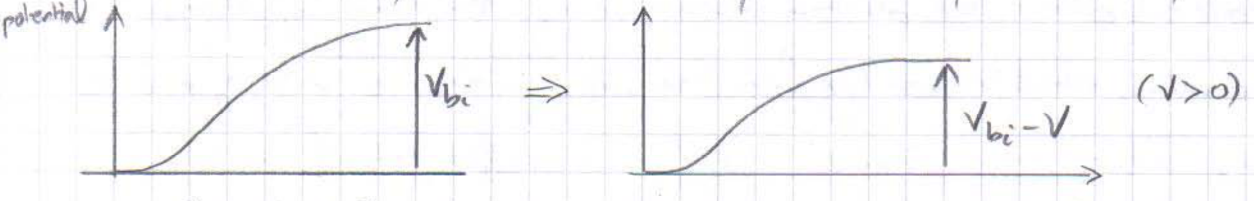


As it is assumed that $G_n \tau_n \gg n_0$ and $G_p \tau_p \gg p_0$, quasi-Fermi levels do not coincide with each other and are even repelled from the Fermi level E_F by the amount:

$$\boxed{(E_{F_n} - E_F) - (E_{F_p} - E_F) = k_B T \ln\left[\left(\frac{G_n \tau_n}{n_0}\right) \Big/ \left(\frac{G_p \tau_p}{p_0}\right)\right]}$$

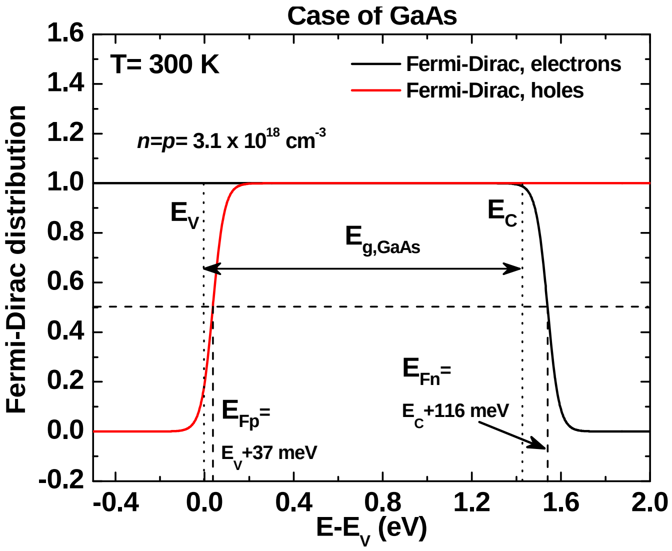
Evolution of the width of the depletion region $W(V)$ when $V>0$, $V<0$. Microscopic description in the ideal p-n junction case. Origin of the saturation current under reverse bias.

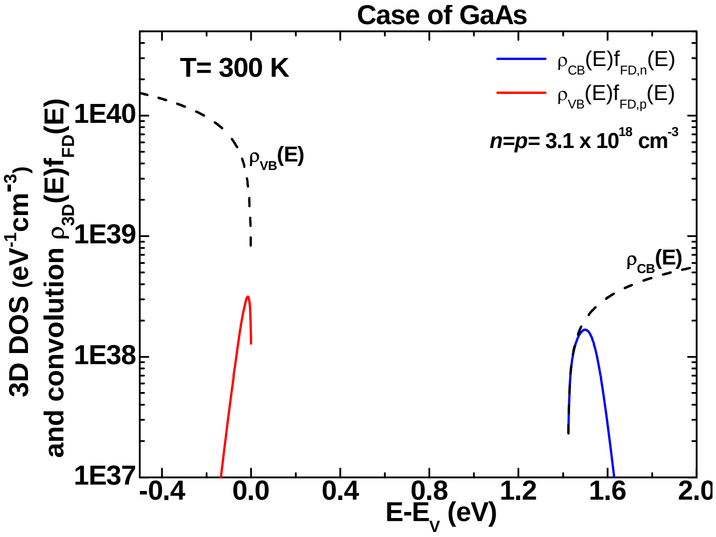
When V increases, the electrostatic potential drop in the depletion region decreases.

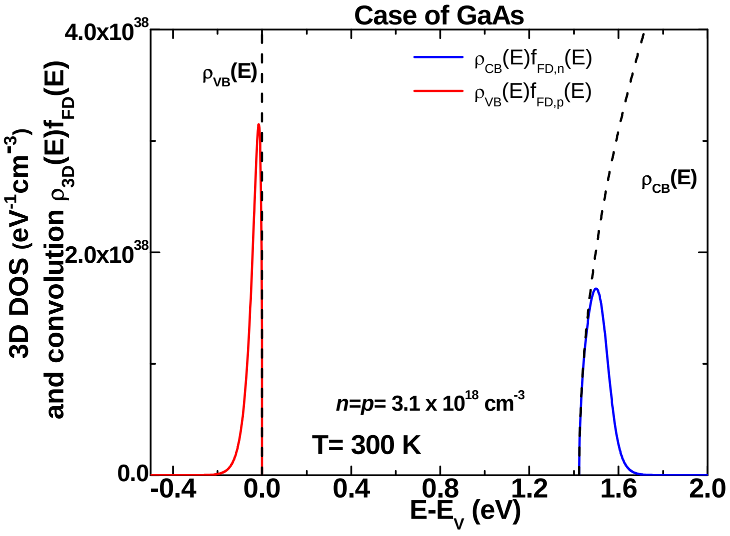


As the electric field is linked to the gradient of the electrostatic potential drop, we deduce that when $V>0$, $|\vec{E}| \searrow$. It means that the extent of the

Figures relative to quasi-Fermi levels (case of GaAs)







depletion region will be altered and in this peculiar case it decreases.

As a result under forward bias the equilibrium between drift and diffusion currents for a given type of carriers is broken. As the electric field decreases, the diffusion current is promoted/favored so that the number of carriers going from a region where they are majority carriers to a region where they are minority ones will increase. Conversely, when $V<0$ $|\vec{E}|\nearrow$ and $W(V)\nearrow$ so that the drift currents will dominate over the diffusion ones.

The diffusion current depends exponentially on the applied voltage V. As an illustration, the number of electrons injected on the p-side compared with thermal equilibrium is $n_p - n_{p_0} = n_{p_0}[\exp(eV/k_BT)-1]$. In the limit where $eV \gg k_BT$ (~ 25 meV at 300K), $n_p - n_{p_0} \approx n_{p_0}\exp(eV/k_BT)$.

Conversely, under reverse bias the number of carriers which can overcome the electrostatic potential will exponentially decrease so that beyond a certain value of V the diffusion will be limited and only the drift current will contribute at the junction level. The latter is in fact governed by the generation-recombination term of minority carriers within the limit of one diffusion length with respect to the boundaries of the depletion region. If one considers an electron-hole pair generated on the p-side at a distance $L < L_n$ of the position $-x_p$, the electon will diffuse toward the depletion region and will then be swept from $-x_p$ to x_n by the electric field. For obvious symmetrical reasons a similar physical phenomenon will apply to holes on the n-side of the junction.

Note that the drift current will rather depend on the carrier generation rate than on the value of the electric field (Once donor and acceptor states are fully ionized the dependence of $W(V)$ to the temperature will be much smaller than that of n_{p_0} and p_{n_0}). As a result as a first approximation we can write:

$$J_{diffusion} = J_S \exp(eV/k_BT) \quad \text{and} \quad J_{drift} = -J_S$$

$$J = J_{diffusion} + J_{drift} = J_S[\exp(eV/k_BT) - 1] \text{ and when } V=0$$

$$J_{diffusion} = J_S = -J_{drift} \text{ and thus } J = 0.$$